More three-point correlators of giant magnons with finite size

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Abstract

In the framework of the semiclassical approach, we compute the normalized structure constants in three-point correlation functions, when two of the vertex operators correspond to heavy string states, while the third vertex corresponds to a light state. This is done for the case when the heavy string states are *finite-size* giant magnons with one or two angular momenta, and for two different choices of the light state, corresponding to dilaton operator and primary scalar operator. The relevant operators in the dual gauge theory are $Tr\left(F_{\mu\nu}^2\, Z^j + \ldots\right)$ and $Tr\left(Z^j\right)$. We first consider the case of $AdS_5 \times S^5$ and $\mathcal{N}=4$ super Yang-Mills. Then we extend the obtained results to the γ-deformed $AdS_5 \times S^5_\gamma$, dual to $\mathcal{N}=1$ super Yang-Mills theory, arising as an exactly marginal deformation of $\mathcal{N}=4$ super Yang-Mills.

1 Introduction

It is known that the correlation functions of any conformal field theory can be determined in principle in terms of the basic conformal data $\{\Delta_i, C_{ijk}\}$, where Δ_i are the conformal dimensions defined by the two-point correlation functions

$$\left\langle \mathcal{O}_i^\dagger(x_1)\mathcal{O}_j(x_2)\right\rangle = \frac{C_{12}\delta_{ij}}{|x_1 - x_2|^{2\Delta_i}}$$

and C_{ijk} are the structure constants in the operator product expansion

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\rangle = \frac{C_{ijk}}{|x_1 - x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1 - x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2 - x_3|^{\Delta_2+\Delta_3-\Delta_1}}.$$

Therefore, the determination of the initial conformal data for a given conformal field theory is the most important step in the conformal bootstrap approach. While this is well-established in two dimensions where the conformal symmetry is infinite dimensional [1], it is extremely difficult to extend the procedure to higher dimensional space-times.

The AdS/CFT correspondence [2] between type IIB string theory on $\mathrm{AdS}_5 \times S^5$ space and $\mathcal{N} = 4$ super Yang-Mills theory (SYM) in four dimensions, has provided a most promising framework. A lot of impressive results have been obtained in this field based on the integrability discovered in the planar limit of the $\mathcal{N} = 4$ SYM. In particular, the thermodynamic Bethe ansatz approach based on non-perturbative worldsheet S-matrix has been formulated to provide the conformal dimensions of SYM operators with arbitrary number of elementary fields for generic value of 't Hooft coupling constant λ (for a recent review see [3]). In the strong coupling limit $\lambda \gg 1$, the AdS/CFT duality relates the conformal dimensions to the energy and angular momenta of certain classical string configurations.

The situation is similar for the γ-deformed (or TsT-transformed) case, namely duality between string theory on $AdS_5 \times S^5_\gamma$ background, dual to $\mathcal{N} = 1$ SYM, arising as marginal deformation of $\mathcal{N} = 4$ SYM [4, 5].

There have been many interesting achievements on three-point correlation functions in the AdS/CFT context. Three-point functions for chiral primary operators have been computed first in the AdS_5 supergravity approximation [6]. Recently, several interesting developments have been made by considering general heavy string states. An efficient method to compute two-point correlation functions in the strong coupling limit is to evaluate string partition function for a heavy string state propagating in the AdS space between two boundary points based on a path integral method [7, 8]. This method has been extended to the three-point functions of two heavy string states and a light supergravity mode [9, 10, 11]. Relying on these achievements, many interesting results concerning three-point correlators of two heavy modes and one light mode have been obtained [9]-[29].

The three-point functions of two heavy operators and a light operator can be approximated by a supergravity vertex operator evaluated at the heavy classical string configuration:

$$\langle V_H(x_1)V_H(x_2)V_L(x_3)\rangle = V_L(x_3)_{\text{classical}}.$$

For $|x_1| = |x_2| = 1$, $x_3 = 0$, the correlation function reduces to

$$\langle V_{H_1}(x_1) V_{H_2}(x_2) V_L(0) \rangle = \frac{C_{123}}{|x_1 - x_2|^{2\Delta_H}}.$$

Then, the normalized structure constants

$$\mathcal{C}_3 = \frac{C_{123}}{C_{12}}$$

can be found from

$$\mathcal{C}_3 = c_\Delta V_L(0)_{\text{classical}}, \tag{1.1}$$

were c_Δ is the normalized constant of the corresponding light vertex operator.

Very recently, first results describing finite-size effects on the three-point correlators appeared [24, 25, 28]. In this note we are going to obtain more three-point correlation functions for the cases when the heavy string states are *finite-size* giant magnons with one or two angular momenta, and for two different choices of the light state, corresponding to dilaton operator and primary scalar operator. We first consider the case of giant magnons in $AdS_5 \times S^5$, and then we extend the obtained results to giant magnons on γ-deformed $AdS_5 \times S^5_\gamma$ background.

2 $AdS_5 \times S^5$ and $\mathcal{N} = 4$ SYM

Let us first recall the relations between the coordinates, which we are going to use further on. If we denote the string embedding coordinates on AdS and sphere parts of the $AdS_5 \times S^5$ background with Y and X respectively, then

$$Y_1 + iY_2 = \sinh\rho\ \sin\eta\ e^{i\varphi_1}, \quad Y_3 + iY_4 = \sinh\rho\ \cos\eta\ e^{i\varphi_2}, \quad Y_5 + iY_0 = \cosh\rho\ e^{it},$$

are related to the Poincare coordinates by

$$Y_m = \frac{x_m}{z}, \quad Y_4 = \frac{1}{2z}\left(x^m x_m + z^2 - 1\right), \quad Y_5 = \frac{1}{2z}\left(x^m x_m + z^2 + 1\right),$$

where $x^m x_m = -x_0^2 + x_i x_i$, with $m = 0, 1, 2, 3$ and $i = 1, 2, 3$.

Since we are going to compute the three-point correlators containing two heavy operators corresponding to dyonic giant magnons, we restrict ourselves to the $R_t \times S^3$ subspace of $AdS_5 \times S^5$. In that case, we can explore the reduction of the string dynamics to the Neumann-Rosochatius (NR) integrable system by using the ansatz [30]

$$\begin{aligned} &t(\tau,\sigma) = \kappa\tau, \quad \theta(\tau,\sigma) = \theta(\xi), \quad \phi_j(\tau,\sigma) = \omega_j\tau + f_j(\xi), \\ &\xi = \alpha\sigma + \beta\tau, \quad \kappa, \omega_j, \alpha, \beta = constants, \quad j = 1, 2. \end{aligned} \tag{2.1}$$

Then the string Lagrangian in conformal gauge, on the three-sphere, can be written as (prime is used for $d/d\xi$)

$$\mathcal{L}_{S^3} = (\alpha^2-\beta^2)\left[\theta'^2 + \sin^2\theta\left(f_1' - \frac{\beta\omega_1}{\alpha^2-\beta^2}\right)^2 + \cos^2\theta\left(f_2' - \frac{\beta\omega_2}{\alpha^2-\beta^2}\right)^2 \right.$$
$$\left. - \frac{\alpha^2}{(\alpha^2-\beta^2)^2}\left(\omega_1^2\sin^2\theta + \omega_2^2\cos^2\theta\right)\right]. \tag{2.2}$$

One can show that the first integrals of the equations of motion for $f_j(\xi)$, $\theta(\xi)$, take the form

$$\begin{aligned} f_1' &= \frac{\omega_1}{\alpha}\frac{v}{1-v^2}\left(\frac{W}{1-\chi} - 1\right), \\ f_2' &= -\frac{\omega_1}{\alpha}\frac{uv}{1-v^2}, \\ \theta' &= \frac{\omega_1}{\alpha}\frac{\sqrt{1-u^2}}{1-v^2}\sqrt{\frac{(\chi_p-\chi)(\chi-\chi_m)}{1-\chi}}, \end{aligned} \tag{2.3}$$

where

$$\begin{aligned} \chi_p + \chi_m &= \frac{2-(1+v^2)W-u^2}{1-u^2}, \\ \chi_p\chi_m &= \frac{1-(1+v^2)W+(vW)^2}{1-u^2}. \end{aligned} \tag{2.4}$$

The case of finite-size giant magnons, corresponds to

$$0 < \chi_m < \chi < \chi_p < 1.$$

In (2.3) and (2.4) the following notations have been used

$$\chi = \cos^2\theta, \quad v = -\frac{\beta}{\alpha}, \quad u = \frac{\omega_2}{\omega_1}, \quad W = \left(\frac{\kappa}{\omega_1}\right)^2.$$

The replacement into (2.2) gives (we set for simplicity $\alpha = \omega_1 = 1$)

$$\mathcal{L}^{gm}_{S^3} = -\frac{1}{1-v^2}\left[2-(1+v^2)W - 2\left(1-u^2\right)\chi\right]. \tag{2.5}$$

Let us also note that the first integral for χ is given by

$$\chi' = \frac{2\sqrt{1-u^2}}{1-v^2}\sqrt{\chi(\chi_p-\chi)(\chi-\chi_m)}. \tag{2.6}$$

2.1 Dilaton operator

For the dilaton vertex we have [11]

$$V^d = (Y_4 + Y_5)^{-\Delta_d} (X_1 + iX_2)^j \left[z^{-2} \left(\partial_+ x_m \partial_- x^m + \partial_+ z \partial_- z\right) + \partial_+ X_k \partial_- X_k\right], \tag{2.7}$$

where the scaling dimension $\Delta_d = 4 + j$ to the leading order in the large $\sqrt{\lambda}$ expansion. The corresponding operator in the dual gauge theory should be proportional to $Tr\left(F^2_{\mu\nu}\ Z^j + \ldots\right)$, or for $j = 0$, just to the SYM Lagrangian.

The normalized structure constant (1.1) can be computed by using (2.7), applied for the case of giant magnons, to be [21, 24] ($i\tau = \tau_e$)

$$\mathcal{C}_3 = c^d_\Delta \int_{-\infty}^{\infty} \frac{d\tau_e}{\cosh^{4+j}(\sqrt{W}\tau_e)} \int_{-L}^{L} d\sigma \left(W + \mathcal{L}^{gm}_{S^3}\right). \tag{2.8}$$

Here, the parameter L is introduced to take into account the *finite-size* of the giant magnons. The normalization constant of the dilaton vertex operator c^d_Δ is given by [35, 9]

$$c^d_\Delta = \frac{\sqrt{\lambda}}{128\pi N} \frac{\sqrt{(j+1)(j+2)(j+3)}}{2^j}.$$

The integration over τ_e in (2.8) gives

$$\int_{-\infty}^{\infty} \frac{d\tau_e}{\cosh^{4+j}(\sqrt{W}\tau_e)} = \sqrt{\frac{\pi}{W}} \frac{\Gamma\left(\frac{4+j}{2}\right)}{\Gamma\left(\frac{5+j}{2}\right)}.$$

The integration over σ can be replaced by integration over χ according to

$$\int_{-L}^{L} d\sigma = 2 \int_{\chi_m}^{\chi_p} \frac{d\chi}{\chi'}, \tag{2.9}$$

where χ' is given in (2.6). As a result (2.8) becomes

$$\begin{aligned} \mathcal{C}_3 = 2\pi^{3/2} c^d_\Delta \frac{\Gamma\left(\frac{4+j}{2}\right)}{\Gamma\left(\frac{5+j}{2}\right)} \frac{\chi_p^{\frac{j-1}{2}}}{\sqrt{(1-u^2)W}} \\ \left[(1-u^2)\chi_p\ {}_2F_1\left(\frac{1}{2}, -\frac{1}{2} - \frac{j}{2}; 1; 1 - \frac{\chi_m}{\chi_p}\right)\right. \\ \left. - (1-W)\ {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - \frac{j}{2}; 1; 1 - \frac{\chi_m}{\chi_p}\right)\right]. \end{aligned} \tag{2.10}$$

A few comments are in order. The structure constant in (2.10) corresponds to finite-size dyonic giant magnons, i.e. with two angular momenta. The case of finite-size giant magnons with one angular momentum nonzero can be obtained by setting $u = 0$ (χ_p, χ_m also depend on u according to (2.4)). The infinite size case [21] is reproduced for $W = 1$, $\chi_m = 0$. For $j = 0$, (2.10) reduces to the result of [24].

2.2 Primary scalar operator

The primary scalar vertex is [35, 9, 11]

$$V^{pr} = (Y_4 + Y_5)^{-\Delta_{pr}} (X_1 + iX_2)^j \left[z^{-2} \left(\partial_+ x_m \partial_- x^m - \partial_+ z \partial_- z\right) - \partial_+ X_k \partial_- X_k\right], \tag{2.11}$$

where now the scaling dimension is $\Delta_{pr} = j$. The corresponding operator in the dual gauge theory is $Tr\left(Z^j\right)$.

For giant magnons we have [21]

$$z^{-2} \left(\partial_+ x_m \partial_- x^m - \partial_+ z \partial_- z\right) = \kappa^2 \left(\frac{2}{\cosh^2(\kappa\tau_e)} - 1\right).$$

Then the light vertex operator becomes

$$V^{pr} = \frac{\cos^j \theta}{\cosh^j(\kappa\tau_e)} \left[\kappa^2 \left(\frac{2}{\cosh^2(\kappa\tau_e)} - 1\right) - \mathcal{L}^{gm}_{S^3}\right],$$

where the infinite-size case was considered in [21], while for the finite-size giant magnons $\mathcal{L}^{gm}_{S^3}$ should be taken from (2.5). As a consequence, the normalized structure constant in the corresponding three-point function, for the case under consideration, takes the form:

$$\begin{aligned} \mathcal{C}^{pr}_3 \;\; &= \;\; c^{pr}_{\Delta} \left[\int_{-\infty}^{\infty} d\tau_e \frac{W}{\cosh^j(\sqrt{W}\tau_e)} \left(\frac{2}{\cosh^2(\sqrt{W}\tau_e)} - 1\right) \int_{-L}^{L} d\sigma \chi^{\frac{j}{2}} \right. \\ &\left. - \int_{-\infty}^{\infty} \frac{d\tau_e}{\cosh^j(\sqrt{W}\tau_e)} \int_{-L}^{L} d\sigma \chi^{\frac{j}{2}} \mathcal{L}^{gm}_{S^3}\right]. \end{aligned} \tag{2.12}$$

Performing the integrations in (2.12), by using (2.9), (2.6), one finally finds

$$\begin{aligned} \mathcal{C}^{pr}_3 &= \pi^{3/2} c^{pr}_{\Delta} \frac{\Gamma\left(\frac{j}{2}\right)}{\Gamma\left(\frac{3+j}{2}\right)} \frac{\chi_p^{\frac{j-1}{2}}}{\sqrt{(1-u^2)W}} \\ &\left[\left(1 - W + j(1 - v^2 W)\right) \,{}_2F_1\left(\frac{1}{2}, \frac{1}{2} - \frac{j}{2}; 1; 1 - \frac{\chi_m}{\chi_p}\right)\right. \\ &\left. - (1+j)\left(1-u^2\right)\chi_p \,{}_2F_1\left(\frac{1}{2}, -\frac{1}{2} - \frac{j}{2}; 1; 1 - \frac{\chi_m}{\chi_p}\right)\right]. \end{aligned} \tag{2.13}$$

As before, the case of finite-size giant magnons with one angular momentum can be obtained by setting $u = 0$ in (2.13).

It was observed in [21] that for the infinite-size case, $\mathcal{C}^{pr}_3$ is nonzero only for giant magnons with two angular momenta. For giant magnons with one angular momentum $\mathcal{C}^{pr}_3$ vanishes. Let us see if we can reproduce this result from (2.13). To this end, we fix $W = 1$, $\chi_m = 0$. Then according to (2.4), χ_p becomes

$$\chi_p = \frac{1 - v^2 - u^2}{1 - u^2},$$

and one can show that $\mathcal{C}_3^{pr}$ reduces to

$$\mathcal{C}_{3\infty}^{pr} = 2\pi c_\Delta^{pr} \frac{\Gamma\left(\frac{j}{2}\right)\Gamma\left(1+\frac{j}{2}\right)}{\Gamma\left(\frac{3+j}{2}\right)\Gamma\left(\frac{1+j}{2}\right)} \frac{u^2}{\sqrt{1-u^2-v^2}} \left(\frac{1-v^2-u^2}{1-u^2}\right)^{j/2}. \tag{2.14}$$

Since we know that the case of giant magnons with one angular momentum corresponds to $u=0$, (2.14) confirms the observation made in [21].

It is also interesting to see how (2.14) looks written in terms of the second angular momentum J_2 and the worldsheet momentum p of the string, where p is identified with the magnon momentum in the dual spin chain on the field theory side. For this purpose, we have to know the relations between (v,u) and (J_2,p). One way to find them is to look at the finite-size expansions of J_2, p, and to take into account only the leading terms there, because at the moment we are considering the infinite-size case. According to [36], this gives

$$v = \frac{\sin(p)}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}}, \quad u = \frac{\mathcal{J}_2}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}}, \tag{2.15}$$

where

$$\mathcal{J}_2 = \frac{2\pi J_2}{\sqrt{\lambda}}.$$

Replacing (2.15) into (2.14), one ends up with the following expression for $\mathcal{C}_{3\infty}^{pr}$

$$\mathcal{C}_{3\infty}^{pr} = \pi c_\Delta^{pr} \frac{\Gamma\left(\frac{j}{2}\right)\Gamma\left(1+\frac{j}{2}\right)}{\Gamma\left(\frac{3+j}{2}\right)\Gamma\left(\frac{1+j}{2}\right)} \mathcal{J}_2^2 \frac{\sin^{j-2}(p/2)}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}}. \tag{2.16}$$

3 $AdS_5 \times S_\gamma^5$ and $\mathcal{N}=1$ SYM

Investigations on AdS/CFT duality [2] for the cases with reduced or without supersymmetry is of obvious interest and importance. An interesting example of such correspondence between gauge and string theory models with reduced supersymmetry is provided by an exactly marginal deformation of $\mathcal{N}=4$ super Yang-Mills theory [37] and string theory on a β-deformed $AdS_5 \times S^5$ background suggested by Lunin and Maldacena in [4]. When $\beta \equiv \gamma$ is real, the deformed background can be obtained from $AdS_5 \times S^5$ by the so-called TsT transformation. It includes T-duality on one angle variable, a shift of another isometry variable, then a second T-duality on the first angle [5].

An essential property of the TsT transformation is that it preserves the classical integrability of string theory on $AdS_5 \times S^5$ [5]. The γ-dependence enters only through the *twisted* boundary conditions and the *level-matching* condition. The last one is modified since a closed string in the deformed background corresponds to an open string on $AdS_5 \times S^5$ in general.

The parameter $\tilde{\gamma}$, which appears in the string action, is related to the deformation parameter γ as

$$\tilde{\gamma} = \sqrt{\lambda}\,\gamma.$$

The effect of introducing γ on the field theory side of the duality is to modify the super potential as follows

$$W \propto tr\left(e^{i\pi\gamma}\Phi_1\Phi_2\Phi_3 - e^{-i\pi\gamma}\Phi_1\Phi_3\Phi_2\right).$$

This leads to reduction of the supersymmetry of the SYM theory from $\mathcal{N} = 4$ to $\mathcal{N} = 1$.

Since we are going to consider three-point correlation functions with two vertices corresponding to dyonic giant magnon states, we can restrict ourselves to the subspace $R_t \times S^3_\gamma$ of $AdS_5 \times S^5_\gamma$ background. Then one can show that by using the ansatz (2.1), the string Lagrangian in conformal gauge, on the γ-deformed three-sphere S^3_γ, can be written as [28]

$$\begin{aligned}\mathcal{L}_\gamma = (\alpha^2-\beta^2)\Bigg[\theta'^2 &+ G\sin^2\theta\left(f_1' - \frac{\beta\omega_1}{\alpha^2-\beta^2}\right)^2 + G\cos^2\theta\left(f_2' - \frac{\beta\omega_2}{\alpha^2-\beta^2}\right)^2 \\ &- \frac{\alpha^2}{(\alpha^2-\beta^2)^2}G\left(\omega_1^2\sin^2\theta + \omega_2^2\cos^2\theta\right) + 2\alpha\tilde{\gamma}G\sin^2\theta\cos^2\theta\frac{\omega_2 f_1' - \omega_1 f_2'}{\alpha^2-\beta^2}\Bigg], \qquad (3.1)\end{aligned}$$

where

$$G = \frac{1}{1+\tilde{\gamma}^2\sin^2\theta\cos^2\theta}.$$

In full analogy with the undeformed case, by using (3.1) and the Virasoro constraints, one can find the following first integrals

$$\begin{aligned} f_1' &= \frac{\Omega_1}{\alpha}\frac{1}{1-v^2}\left[\frac{vW-uK}{1-\chi} - v(1-\tilde{\gamma}K) - \tilde{\gamma}u\chi\right], \\ f_2' &= \frac{\Omega_1}{\alpha}\frac{1}{1-v^2}\left[\frac{K}{\chi} - uv(1-\tilde{\gamma}K) - \tilde{\gamma}v^2W + \tilde{\gamma}(1-\chi)\right], \qquad (3.2) \\ \theta' &= \frac{\Omega_1}{\alpha}\frac{\sqrt{1-u^2}}{1-v^2}\sqrt{\frac{(\chi_p-\chi)(\chi-\chi_m)(\chi-\chi_n)}{\chi(1-\chi)}}, \end{aligned}$$

where

$$\begin{aligned} \chi_p + \chi_m + \chi_n &= \frac{2-(1+v^2)W-u^2}{1-u^2}, \\ \chi_p\chi_m + \chi_p\chi_n + \chi_m\chi_n &= \frac{1-(1+v^2)W+(vW-uK)^2-K^2}{1-u^2}, \qquad (3.3) \\ \chi_p\chi_m\chi_n &= -\frac{K^2}{1-u^2}. \end{aligned}$$

Now

$$u = \frac{\Omega_2}{\Omega_1}, \quad W = \left(\frac{\kappa}{\Omega_1}\right)^2, \quad K = \frac{C_2}{\alpha\Omega_1},$$
$$\Omega_1 = \omega_1\left(1 + \tilde{\gamma}\frac{C_2}{\alpha\omega_1}\right), \quad \Omega_2 = \omega_2\left(1 - \tilde{\gamma}\frac{C_1}{\alpha\omega_2}\right),$$

while χ and v are the same as before: $\chi = \cos^2\theta$, $v = -\beta/\alpha$. The case of finite-size giant magnons corresponds to

$$0 < \chi_m < \chi < \chi_p < 1, \quad \chi_n < 0.$$

Replacing (3.2) and (3.3) into (3.1), we find the final form of the Lagrangian to be (we fix $\alpha = \Omega_1 = 1$)

$$\mathcal{L}_{\tilde{\gamma}} = -\frac{1}{1-v^2}\left[2 - (1+v^2)W - 2\tilde{\gamma}K - 2\left(1 - \tilde{\gamma}K - u\left(u - \tilde{\gamma}uK + \tilde{\gamma}vW\right)\right)\chi\right]. \quad (3.4)$$

After setting $\tilde{\gamma} = 0$ in (3.4) it coincides with (2.5) as it should be. Now, the first integral for χ reads

$$\chi' = \frac{2\sqrt{1-u^2}}{1-v^2}\sqrt{(\chi_p - \chi)(\chi - \chi_m)(\chi - \chi_n)}. \quad (3.5)$$

3.1 Dilaton operator

To compute the normalized structure constant (1.1) for the case of two finite-size dyonic giant magnons living on the γ-deformed three-sphere S^3_γ, we have to modify (2.8) by using (3.4)

$$\mathcal{C}_3 \to \mathcal{C}_3^\gamma = c^d_\Delta \int_{-\infty}^{\infty} \frac{d\tau_e}{\cosh^{4+j}(\sqrt{W}\tau_e)} \int_{-L}^{L} d\sigma \left(W + \mathcal{L}_{\tilde{\gamma}}\right). \quad (3.6)$$

Replacing the integration over σ according to (2.9) and using (3.5), one finds

$$\mathcal{C}_3^\gamma = 2\pi^{3/2} c^d_\Delta \frac{\Gamma\left(\frac{4+j}{2}\right)}{\Gamma\left(\frac{5+j}{2}\right)} \frac{\chi_p^{j/2}}{\sqrt{(1-u^2)W\left(\chi_p - \chi_n\right)}} \quad (3.7)$$
$$\left\{\left[1 - \tilde{\gamma}K - u\left(u + \tilde{\gamma}(vW - uK)\right)\right]\chi_p F_1\left(1/2, 1/2, -1-j/2; 1; 1-\epsilon, 1 - \frac{\chi_m}{\chi_p}\right)\right.$$
$$\left. - \left(1 - W - \tilde{\gamma}K\right) F_1\left(1/2, 1/2, -j/2; 1; 1-\epsilon, 1 - \frac{\chi_m}{\chi_p}\right)\right\},$$

where

$$\epsilon = \frac{\chi_m - \chi_n}{\chi_p - \chi_n}, \quad (3.8)$$

and $F_1(a,b_1,b_2;c;z_1,z_2)$ is one of the hypergeometric functions of two variables ($Appell F_1$). In writing (3.7), we used the following property of $F_1(a,b_1,b_2;c;z_1,z_2)$ [38]

$$F_1(a,b_1,b_2;c;z_1,z_2) = (1-z_1)^{-b_1}(1-z_2)^{-b_2} F_1\left(c-a,b_1,b_2;c;\frac{z_1}{z_1-1},\frac{z_2}{z_2-1}\right).$$

Thus, the arguments $(1-\epsilon, 1-\chi_m/\chi_p) \in (0,1)$. This representation gives the possibility to use the defining series for $F_1(a,b_1,b_2;c;z_1,z_2)$,

$$F_1(a,b_1,b_2;c;z_1,z_2) = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} \frac{(a)_{k_1+k_2}(b_1)_{k_1}(b_2)_{k_2}}{(c)_{k_1+k_2}} \frac{z_1^{k_1} z_2^{k_2}}{k_1!\ k_2!}, \qquad |z_{1,2}|<1,$$

in order to consider the limits $\epsilon \to 0$, $\chi_m/\chi_p \to 0$, or both. The small ϵ limit corresponds to considering the leading finite-size effect, while $\epsilon = 0$, $\chi_m = 0$, $\chi_n = 0$, $K = 0$, $W = 1$, describes the infinite-size case.

3.2 Primary scalar operator

Since the γ-deformation affects only the sphere, to compute the three-point correlator of two finite-size giant magnons and the primary scalar operator in the deformed case, we can use (2.12), where $\mathcal{L}^{gm}_{S^3}$ must be replaced with (3.4), i.e.

$$\begin{aligned} \mathcal{C}^{pr}_{3\tilde{\gamma}} &= c^{pr}_{\Delta}\left[\int_{-\infty}^{\infty} d\tau_e \frac{W}{\cosh^j(\sqrt{W}\tau_e)}\left(\frac{2}{\cosh^2(\sqrt{W}\tau_e)}-1\right)\int_{-L}^{L} d\sigma \chi^{\frac{j}{2}} \right. \\ &\left. - \int_{-\infty}^{\infty} \frac{d\tau_e}{\cosh^j(\sqrt{W}\tau_e)} \int_{-L}^{L} d\sigma \chi^{\frac{j}{2}} \mathcal{L}_{\tilde{\gamma}}\right]. \end{aligned} \tag{3.9}$$

Performing the integrations in (3.9) one obtains

$$\begin{aligned} &\mathcal{C}^{pr}_{3\tilde{\gamma}} = \pi^{3/2} c^{pr}_{\Delta} \frac{\Gamma\left(\frac{j}{2}\right)}{\Gamma\left(\frac{1+j}{2}\right)} \frac{(1-v^2)\chi_p^{j/2}}{\sqrt{(1-u^2)(\chi_p-\chi_n)}} \\ &\left\{\left[\sqrt{W}\frac{j-1}{j+1} + \frac{1}{\sqrt{W}(1-v^2)}\left(2-(1+v^2)W - 2\tilde{\gamma}K\right)\right] \right. \\ &\times F_1\left(1/2,1/2,-j/2;1;1-\epsilon,1-\frac{\chi_m}{\chi_p}\right) \\ &- \frac{2}{\sqrt{W}(1-v^2)}\left[1-\tilde{\gamma}K - u\left(u-\tilde{\gamma}uK+\tilde{\gamma}vW\right)\right]\chi_p \\ &\left.\times F_1\left(1/2,1/2,-1-j/2;1;1-\epsilon,1-\frac{\chi_m}{\chi_p}\right)\right\}. \end{aligned} \tag{3.10}$$

It can be shown that (3.10) reduces to the undeformed case (2.13) if we fix

$$\tilde{\gamma} = K = \chi_n = 0 \qquad \Rightarrow \epsilon = \frac{\chi_m}{\chi_p}.$$

This can be done by using the following property of the hypergeometric function F_1

$$F_1(a, b_1, b_2; c; z, z) = {}_2F_1\left(a, b_1 + b_2; c; z\right).$$

For the infinite-size case, (3.10) gives

$$\mathcal{C}^{pr}_{3\tilde{\gamma}\infty} = \pi c^{pr}_{\Delta} \frac{\Gamma\left(\frac{j}{2}\right)\Gamma\left(1+\frac{j}{2}\right)}{\Gamma\left(\frac{3+j}{2}\right)\Gamma\left(\frac{1+j}{2}\right)} \mathcal{J}_2 \frac{\sin^{j-2}(p/2)}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}} \left[\mathcal{J}_2 + \tilde{\gamma}\sin^2(p/2)\sin(p)\right]. \quad (3.11)$$

Obviously, for $\tilde{\gamma} = 0$, (3.11) reduces to (2.16) as it should be. Also, (3.11) shows that the three-point correlator is zero even in the γ-deformed case for $\mathcal{J}_2 = 0$, when the giant magnons are of infinite size.

4 Concluding Remarks

In this paper we extended the recently obtained results concerning finite-size effects on the three-point correlation functions in the framework of the semiclassical approximation, corresponding to the case when the heavy string states are *finite-size* giant magnons with one or two angular momenta [24, 25, 28] in several ways.

First of all, we considered the more general case of dilaton vertex with nonzero Kaluza-Klein momentum $j \neq 0$, for which only the infinite-size case was studied [21]. The investigations on finite-size effects in [24, 25] were restricted to the particular case of $j = 0$. In that case, the coupling is just to the Lagrangian [9, 10, 11], i.e. it corresponds to a marginal deformation of the SYM two-point functions by the Lagrangian.

Next, we generalized the infinite-size result of [21] about two heavy giant magnon states and primary scalar operator to take into account the finite-size effect.

On the third place, we extended the infinite-size result of [27] and the finite-size result of [28], by considering $j \neq 0$.

Finally, we derived the finite-size effect on the normalized structure constant in the three-point correlation function of two heavy giant magnon states and primary scalar operator on the γ-deformed $AdS_5 \times S^5_\gamma$ background. Let us point out that even the infinite-size case was not considered by now.

Obviously, many interesting questions are waiting to be answered in this field of research.

Acknowledgements

This work was supported in part by DO 02-257 grant.

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